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K. NAGAI ET AL  
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Fig. 1.

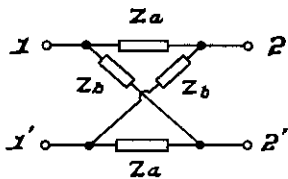


Fig. 2.

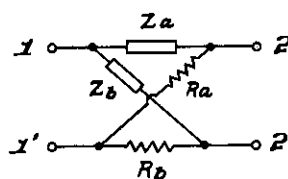


Fig. 3.

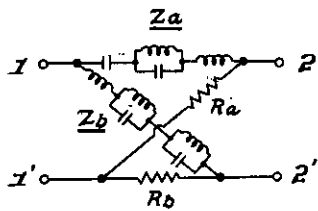


Fig. 4.

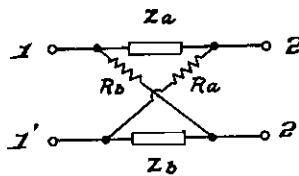


Fig. 5.

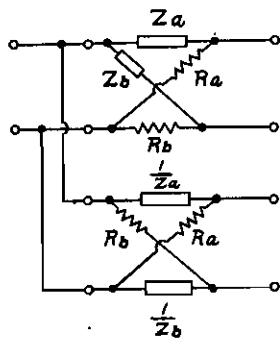
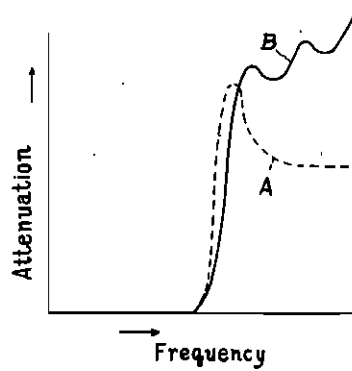


Fig. 6.



INVENTORS  
K. Nagai & S. Uemura  
BY:  
Glascock, Downing & Lebold  
ATTORNEYS.

# ALIEN PROPERTY CUSTODIAN

## FILTERS

Kenzo Nagai and Saburo Uemura, Sendai-shi,  
Japan; vested in the Alien Property Custodian

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This invention relates to filters and more particularly to filters having a lattice type four-terminal network comprising two reactance elements and two resistances in combination and has for its object to provide a voltage filter easier in both the manufacture and the adjustment, but the voltage filter characteristic being substantially equal to that of the symmetrical lattice type filters commonly used.

The common filter heretofore in use is provided by the circuit of a four-terminal network solely comprising reactance elements to transmit the power within a certain range of frequencies, the power beyond the range of frequencies being attenuated or not being transmitted. The resistance component of the circuit is minimized to provide against resistance loss. The operation characteristics and theory in the design of the circuit has fully been studied by Cauer and others, and the circuit has been found theoretically very satisfactory as a filter, but in order to provide a satisfactory attenuation in one section by the use of the circuit, the reactance elements must be manufactured with high accuracy and accurately adjusted in tuning, and consequently, the manufacture of the circuit is considerably difficult and after its completion the adjustment is substantially impossible. Owing to these difficulties, the circuit is not used in general with satisfaction, though it is satisfactory on the theory in the design.

In accordance with the invention there is provided a filter circuit easy in manufacture and adjustment, which is provided by a lattice type four-terminal network comprising two reactance elements and two resistances in combination to be operated with output terminals in nearly opened condition.

The nature of the invention will be more fully understood from the following detailed description and by reference to the accompanying drawings, of which;

Fig. 1 shows a circuit diagram of a filter heretofore in use;

Fig. 2 shows a filter circuit diagram embodying the invention;

Fig. 3 shows a low-pass filter circuit diagram embodying the invention;

Fig. 4 shows an alternative filter circuit diagram;

Fig. 5 shows another alternative filter circuit diagram; and

Fig. 6 shows the frequency-attenuation curves.

In Fig. 1 the conventional filter circuit shown as typical one comprises reactance elements  $Za$ ,

one being inserted between terminals 1 and 2 and the other between other terminals 1' and 2', and other reactance elements  $Zb$ ,  $Zb$ , one being connected across the terminals 1 and 2' and the other across the terminals 2 and 1'. In this case, the reactance elements  $Za$ ,  $Za$  and  $Zb$ ,  $Zb$  must be manufactured and adjusted in tuning with high accuracy, in order to provide a satisfactory attenuation in one section, and consequently the manufacture of the circuit is considerably difficult and after its completion, the adjustment is substantially impossible, as mentioned above.

Coming now to a description of the circuit according to the invention with reference to Fig. 2 which illustrates one embodiment of the invention, the circuit comprises a reactance element  $Za$  inserted between an input terminal 1 and an output terminal 2, another reactance element  $Zb$  connected across the input terminal 1 and another output terminal 2', a resistance  $Ra$  connected across another input terminal 1' and the output terminal 2 and another resistance  $Rb$  inserted between the input and the output terminals 1' and 2' to form a four-terminal network. The resistances  $Ra$  and  $Rb$  may be of any desired values, and now for the sake of simplicity the both value is assumed to be one ohm.

In this circuit, assuming that  $V_1$  represents the voltage impressed across the input terminals 1 and 1', and  $V_2$  represents the voltage across the output terminals 2 and 2', the effective voltage transmission ratio is given by the following formula:

$$\frac{V_2}{V_1} = \frac{Zb - Za}{(Zb + 1)(Za + 1)} \quad (I)$$

In the formula, when

$$\sqrt{ZaZb} = 1 \quad (II)$$

we have:

$$\frac{V_2}{V_1} = \frac{1 - Zb}{1 + Zb} \quad (III)$$

and

$$\left| \frac{V_2}{V_1} \right| = \left| \frac{1 - Zb}{1 + Zb} \right| = 1 \quad (IV)$$

viz., when  $\sqrt{ZaZb} = 1$ , the effective voltage transmission ratio will be unity and the voltage attenuation will be zero. In the formula (I), when

$$\sqrt{\frac{Za}{Zb}} = 1 \quad (V)$$

we have:

$$\frac{V_2}{V_1} = 0 \quad (VI)$$

It follows that the voltage completely attenuates

and consequently the essential in design of the circuit according to the invention resides in approximating the value of  $\sqrt{Z_a Z_b}$  to unity in the voltage transmission band, and in approximating the value of

$$\sqrt{\frac{Z_a}{Z_b}}$$

to unity in the attenuation band, under the consideration of the voltage transmission only and it is apparent that the circuit designed in this manner will bring about a satisfactory voltage filter action.

It will be now seen that the relation between the reactance elements  $Z_a$  and  $Z_b$  in the formulas (II) and (V) bears a resemblance to the relation between the reactance elements  $Z_a$  and  $Z_b$  in the symmetrical lattice-type network (which will hereinafter be referred to as "Cauer circuit") as shown in Fig. 1.

In the Cauer circuit,

$$\left. \begin{aligned} \sqrt{Z_a Z_b} &= Z_0 \\ \sqrt{\frac{Z_a}{Z_b}} &= \tanh \frac{\theta}{2} \end{aligned} \right\} \quad (VII)$$

Where

$Z_0$  is the image impedance.  
 $\theta$  is the attenuation constant.

Cauer observed that it is the indispensable condition to keep the value of  $Z_0$  in the transmission or passing zone at constant and to approximate the value of  $\tanh$

$$\frac{\theta}{2}$$

in the attenuation zone to unity, and to derive the calculating formulas of  $Z_a$  and  $Z_b$  to meet the condition. Such Cauer circuit is, however, the circuit solely comprising the reactance elements  $Z_a$  and  $Z_b$  and is quite distinctive from the circuit according to the invention, which comprises the reactance elements  $Z_a$  and  $Z_b$  and the resistances  $R_a$  and  $R_b$ , the resemblance between both being found only in the relation between the reactances  $Z_a$  and  $Z_b$ .

According to the invention, the values of  $Z_a$  and  $Z_b$  to meet the formulas (II) and (V) are determined from the said calculating formulas derived by Cauer, by taking advantage of the resemblance between the relation between  $Z_a$  and  $Z_b$  in the circuit according to the invention and that between those in the Cauer circuit, and thus determined values of  $Z_a$  and  $Z_b$ , and any desired values of  $R_a$  and  $R_b$ , for example 1 ohm, are used, whereby satisfactory attenuation or voltage filter action is obtained.

When the input impedance of the circuit according to the invention on the side of the input terminals 1 and 1' is represented by  $Z_{11}$ ,

$$Z_{11} = \frac{(1+Z_a)(1+Z_b)}{2+Z_a+Z_b} \quad (VIII)$$

In the transmission zone, substituting in the equation (VIII) the equation (II), that is,

$$\sqrt{Z_a Z_b} = 1$$

we have:

$$Z_{11} = 1 \quad (IX)$$

Thus, in the voltage transmission zone, the input impedance  $Z_{11}$  is 1 ohm and hence constant, independently of frequency.

In the attenuation zone, substituting in the Equation (VIII) the Equation (V), that is,

$$\sqrt{\frac{Z_a}{Z_b}} = 1$$

we have:

$$Z_{11} = \frac{1}{2}(1+Z_a) \quad (X)$$

Thus, in voltage attenuation zone, the input impedance  $Z_{11}$  is not constant, and includes resistances.

In the above, the values of  $R_a$  and  $R_b$  has been assumed to be 1 ohm for simplicity, but in the circuit according to the invention, substituting  $Z_a'$  and  $Z_b'$  derived from the following equations for  $Z_a$  and  $Z_b$  respectively:

$$\left. \begin{aligned} Z_a &= \frac{Z_a'}{R_a} \\ Z_b &= \frac{Z_b'}{R_b} \end{aligned} \right\} \quad (XI)$$

where

$$R_a = R_b = R$$

the transmission characteristics remain unaltered, while the input impedance  $Z_{11}$  alters into  $Z_{11}'$  as given by

$$Z_{11} = \frac{Z_{11}'}{R} \quad (XII)$$

The values of  $R_a$  and  $R_b$  need not, therefore, be 1 ohm, and may be any desired value.

In the circuit according to the invention, it is desired that the value of  $R_a$  is equal to that of  $R_b$ , but  $Z_{11}$  is not substantially affected by a slight difference between  $R_a$  and  $R_b$ , which may be an error in the manufacture. A slight change of  $R_a$  may cause a slight change of the relative value of  $Z_a$  and  $Z_b$  (see the Equations XI). This holds good with  $R_b$ . The attenuation characteristics may be determined by the value of  $(Z_a - Z_b)$ , as seen from the Formula (I), and in the attenuation zone it is desired to be

$$\frac{Z_a}{Z_b} = 1$$

that is,  $Z_a = Z_b$ , but  $Z_a$  can not be approximated to  $Z_b$  as desired in view of the technical skill on the manufacture of coils and condensers. The fact gives rise to the fatal disadvantage in the Cauer circuit solely comprising  $Z_a$  and  $Z_b$ . On the contrary, in the circuit according to the invention, it is apparent from the foregoing that the relative connection of  $Z_a$  and  $Z_b$  or the attenuation characteristics may easily be adjusted by a slight regulation of  $R_a$  and  $R_b$ , and consequently the circuit may easily be manufactured. Fig. 6 illustrates the attenuation characteristics, in which curve A in dotted line represents the undesirable characteristic when there is a slight difference between  $Z_a$  and  $Z_b$  on the manufacture, and curve B represents the improved characteristic when  $R_a$  is slightly regulated.

Fig. 3 illustrates an application of the circuit as shown in Fig. 2 as a low-pass filter (transmission) by way of example.

Fig. 4 illustrates an alternate embodiment of the invention, in which the reactance element  $Z_a$  is inserted between the input and output terminals 1 and 2, the reactance element  $Z_b$  between the other input and output terminals 1' and 2', as series elements, and the resistance  $R_a$  is connected across the output terminal 2 and the input terminal 1', the resistance  $R_b$  across the input terminal 1' and the output terminal 2, as

shunt elements. In this circuit, assuming that  $V_1$  is the voltage impressed across the input terminals 1' and 1 and  $V_2$  is the voltage across the output terminals 2 and 2', we have:

$$\frac{V_2}{V_1} = \frac{1 - Z_a Z_b}{(1 + Z_a)(1 - Z_b)} \quad (XIII)$$

and if

$$\sqrt{V_a V_b} \doteq 1 \quad (XIV)$$

$$\frac{V_2}{V_1} \doteq 0$$

where is attenuation;  
if

$$\sqrt{\frac{V_a}{V_b}} \doteq 1 \quad (XV)$$

$$\left| \frac{V_2}{V_1} \right| \doteq \left| \frac{1 - Z_a}{1 + Z_a} \right| \doteq 1 \quad (XVI)$$

where is transmission zone and is no attenuation, and the filter characteristic is the reversal of that in the circuit as shown in Fig. 2. The input impedance  $Z_{11}$  is the same as that in the circuit as shown in Fig. 2 and given by the formula (VIII), and is constant in the attenuation zone, while it is given by the formula (X) in the transmission zone. This circuit may also be easily manufactured and adjusted similarly to the circuit as shown in Fig. 2, and bring about satisfactory attenuation or voltage filter action.

In the circuit as described above,

$$\frac{1}{Z_a}$$

and

$$\frac{1}{Z_b}$$

may be used respectively in place of  $Z_a$  and  $Z_b$  without altering the transmission characteristics, and in this case the input impedance  $Z_{11}$  is constant in the attenuation zone, and in the transmission zone will be

$$Z_{11} = \frac{1}{2} \left( 1 + \frac{1}{Z_a} \right) \quad (XVII)$$

The circuit shown in Fig. 4 may be connected to the circuit shown in Fig. 2 in parallel as shown in Fig. 5. In this case, the input impedance of the circuit is constant, it being  $\frac{1}{2}$  ohm, in either case  $\sqrt{Z_a Z_b} = 1$ , or

$$\sqrt{\frac{Z_a}{Z_b}} = 1$$

and consequently the circuit shown in Fig. 5 may be used as an analyzer.

With the circuit according to the invention, a slight loss due to the use of the reactance elements is also avoidable, but the resistance component of one of the reactance elements is higher than that of the other reactance element, the resistance necessary for the balancing of the reactance elements may be added to the other reactance element, so that the attenuation characteristics may be prevented from declining. It will be understood that the equivalent circuits of magnetostriction oscillator, crystal oscillator, mechanical oscillator etc. substantially similar to a reactance circuit may be used as the reactance elements of the circuit according to the invention, and that the circuit according to the invention may be utilized as a retardation circuit for voltage.

KENZO NAGAI  
SABURO UEMURA.