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RESILIENT SUSPENSION OF VIBRATING BODIES

Filed Aug. 8, 1939

Serial No. 288,972

4 Sheets-Sheet 1

FIG. 1

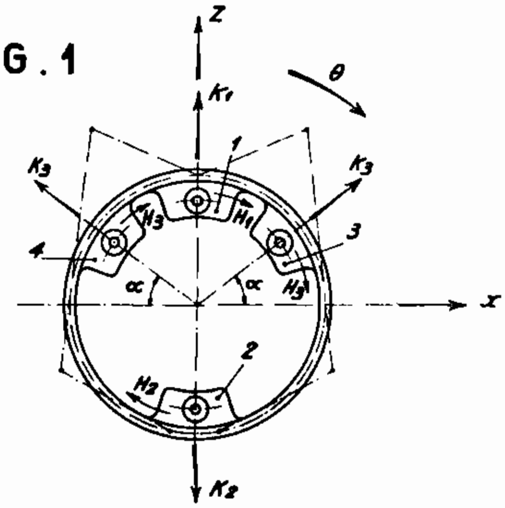

FIG. 2

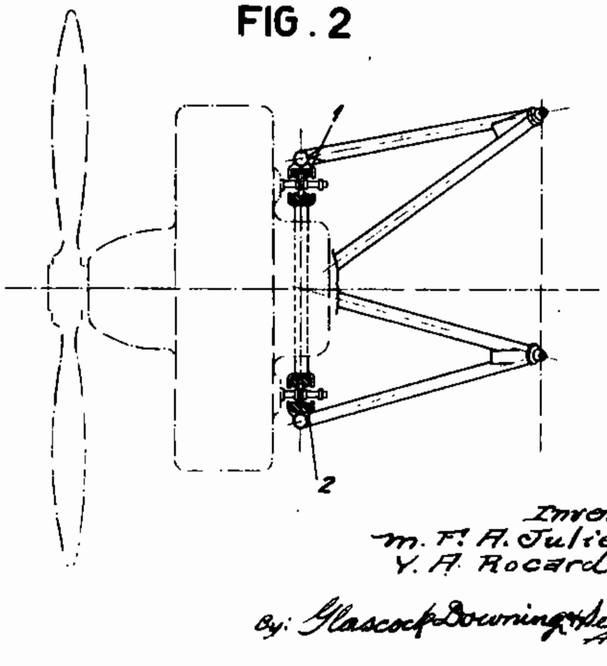

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FIG. 3

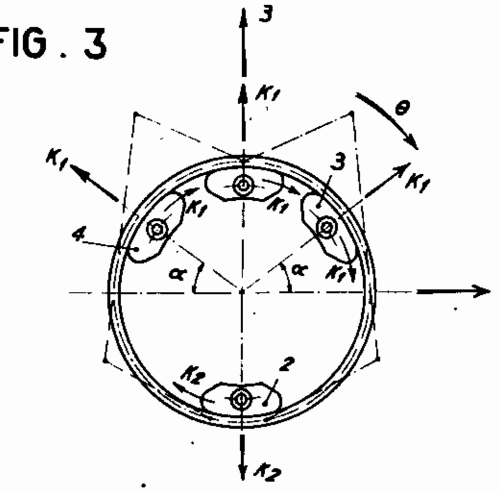

FIG. 4

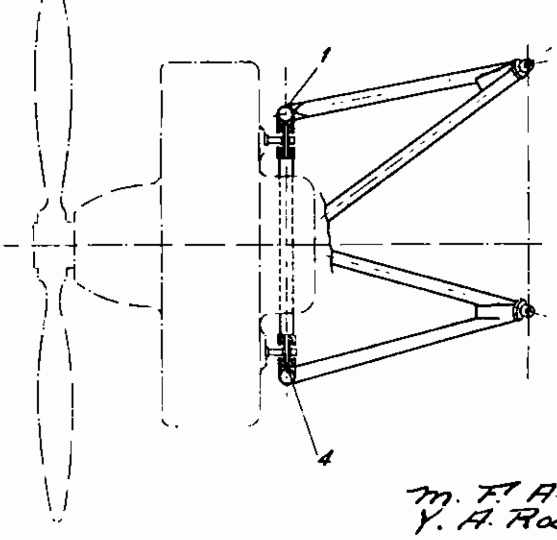

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FIG. 5

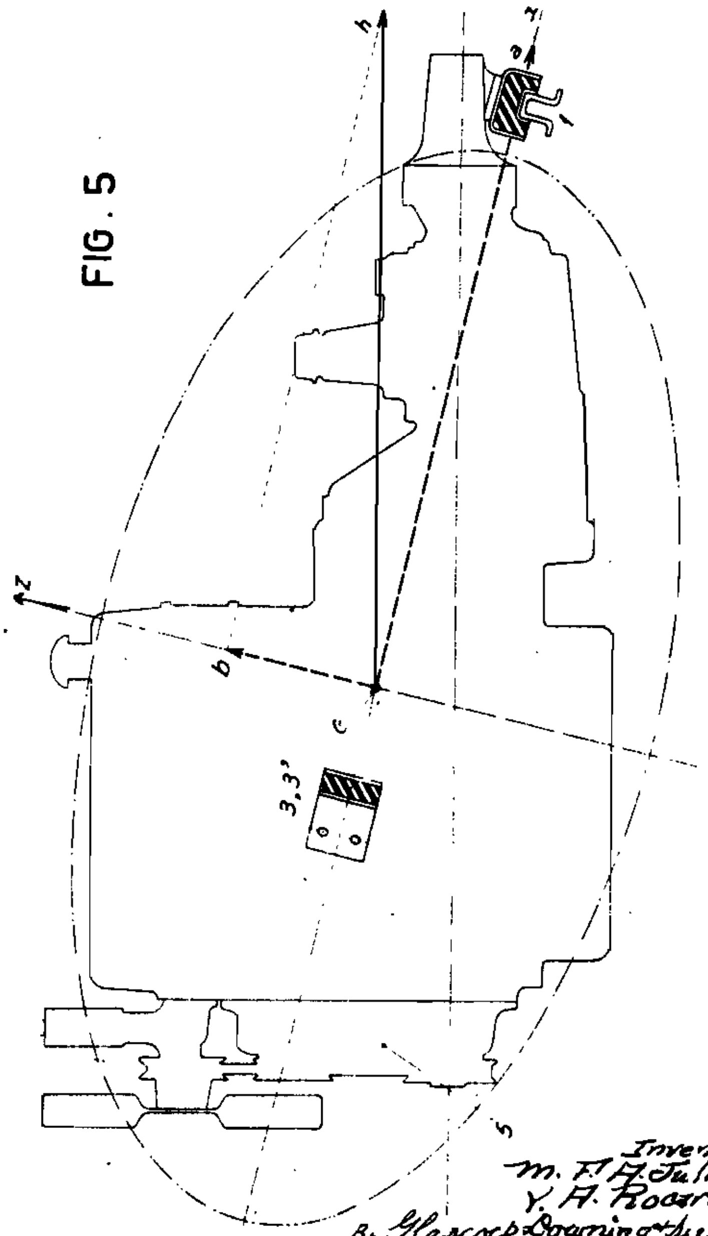

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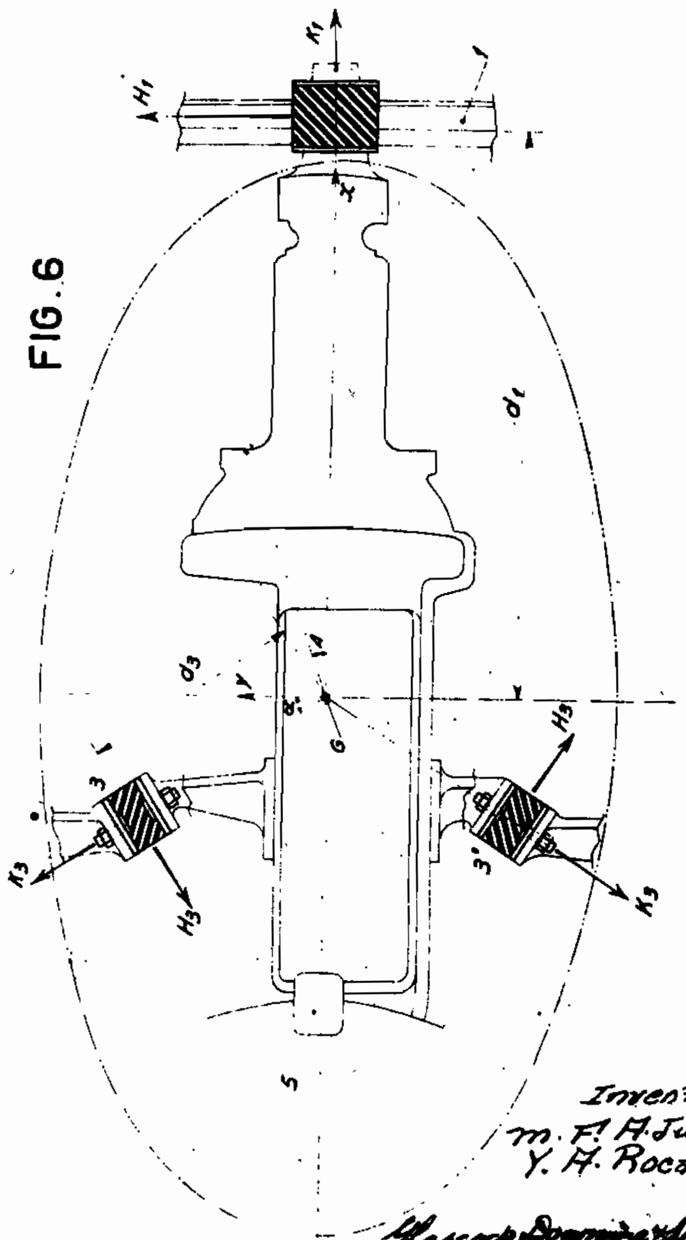
FIG. 6



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ALIEN PROPERTY CUSTODIAN

RESILIENT SUSPENSION OF VIBRATING BODIES

Maurice François Alexandre Julien and Yves André Rocard, Paris, France; vested in the Alien Property Custodian

Application filed August 8, 1939

The present invention relates to improvements in the resilient suspension of vibrating bodies, solid bodies such as engines or the like, for the purpose of eliminating or reducing the phenomena of coupling between the various possible kinds of vibrations.

In order properly to understand the nature of the invention and the means by which it may be performed it is desirable to give certain mathematical developments which have served the applicants as a basis or starting point for the invention.

The position of a vibrating solid body depends on six co-ordinates which are the co-ordinates of position of the centre of gravity, say ox, oy, oz relatively to three axes of co-ordinates in space: ox, oy, oz and three angles θ, ϕ, ψ which characterise the rotation of the body about three rectangular axes (the same as previously in order to simplify the exponent). The main force T of the body has the following general expression, the derivative of x relatively to time being denoted by $\dot{x}$ and the same for $\dot{y}$, $\dot{z}$, $\dot{\theta}$, . . . etc:

$$T=\frac{M}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\frac{i_1\dot{\theta}^2}{2}+\frac{i_2\dot{\phi}^2}{2}+\frac{i_3\dot{\psi}^2}{2}-i_{1.2}\dot{\theta}\dot{\phi}-_{2.3}\dot{\phi}\dot{\psi}-i_{3.1}\dot{\psi}\dot{\theta}$$

m being the mass of the body and i_1, i_2, i_3, $i_{1.2}$, $i_{2.3}$, $i_{3.1}$ being the moments and products of inertia the definition of which is well known.

In the question here contemplated and in which it is merely a matter of vibrations of small amplitudes, the body again finds a fixed position of rest when no vibration is caused. It is then possible to choose the system of rectangular axes which permits the measurement of θ, ϕ, ψ in a manner such that the three products of inertia $i_{1.2}$, $i_{2.3}$, $i_{3.1}$ are zero. The axes are the principal axes of inertia of the solid body; $i_{1.2}$, $i_{2.3}$, $i_{3.1}$ are then the principal moments of inertia of the solid body.

It is convenient to say that the equation:

T=constant

defines in a space having six dimensions where the co-ordinates are: $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$, an ellipsoid with six dimensions referred to its principal axes.

If a solid body is fixed to resilient supports by one of the frames of the latter the other frame being fastened to an assembly of bodies fixed in space, small movements: x, y, z, θ, ϕ, ψ, engender resilient reactions in said supports and it is then possible to calculate as will be shown later in examples, the potential energy brought into action in such a deformation. For a calculation of this kind a knowledge is required of the resilient properties of the supports and of the geometrical arrangement of their mounting; the result will be, however, that for all the movements of small amplitude of x, y, z, θ, ϕ, ψ, that are to be considered, the potential energy of the resilient supports will be of the form:

$$\begin{aligned}V=\;&A_{xx}\frac{x^2}{2}+A_{yy}\frac{y^2}{2}+A_{zz}\frac{z^2}{2}+\\&A_{xy}.xy+A_{yz}.yz+A_{zx}.zx+\\&A_{\theta\theta}\frac{\theta^2}{2}+A_{\phi\phi}\frac{\phi^2}{2}+A_{\psi\psi}\frac{\psi^2}{2}+\\&A_{\theta\phi}.\theta\phi+A_{\phi\psi}.\phi\psi+A_{\psi\theta}.\phi\theta+\\&A_{x\theta}.x\theta+A_{x\phi}.x\phi+A_{x\psi}.x\psi+\\&A_{y\theta}.y\theta+A_{y\phi}.y\phi+A_{y\psi}.y\psi+\\&A_{z\theta}.z\theta+A_{z\phi}.z\phi+A_{z\psi}.z\psi\end{aligned}$$

x, y, z, θ, ϕ, ψ, being naturally counted from their value in the equilibrium position being considered as zero.

It can also be said that:

V=constant

is the equation of an ellipsoid in six dimensional space of co-ordinates x, y, z, θ, ϕ, ψ.

That being so, the equations of the free movement of the suspended body displaced from its equilibrium position are, if η_i is any one of the co-ordinates x, y, . . . etc.

$$\frac{dT}{d\eta_i}+\frac{dV}{d\eta_i}=0$$

(i=1, 2, 3, 4, 5, 6) or, supposing for the sake of simplicity that the fixed axes taken are the principal axes of inertia of the body in its position of rest:

$$m\frac{d^2x}{dt^2}+A_{xx}x+A_{xy}y+A_{xz}z+A_{x\theta}\theta+A_{x\phi}\phi+A_{x\psi}\psi=0$$

$$m\frac{d^2y}{dt^2}+A_{yx}x+A_{yy}y+A_{yz}z+A_{y\theta}\theta+A_{y\phi}\phi+A_{y\psi}\psi=0$$

$$m\frac{d^2z}{dt^2}+A_{zx}x+A_{zy}y+A_{zz}z+A_{z\theta}\theta+A_{z\phi}\phi+A_{z\psi}\psi=0$$

$$i_1\frac{d^2\theta}{dt^2}+A_{\theta x}x+A_{\theta y}y+A_{\theta z}z+A_{\theta\theta}\theta+A_{\theta\phi}\phi+A_{\theta\psi}\psi=0$$

$$i_2\frac{d^2\phi}{dt^2}+A_{\phi x}x+A_{\phi y}y+A_{\phi z}z+A_{\phi\theta}\theta+A_{\phi\phi}\phi+A_{\phi\psi}\psi=0$$

$$i_3\frac{d^2\psi}{dt^2}+A_{\psi x}x+A_{\psi y}y+A_{\phi z}z+A_{\psi\theta}\theta+A_{\psi\phi}\phi+A_{\psi\psi}\psi=0$$

In these equations A_{xy} or A_{yz} has been written indifferently (the same for the terms of the same kind) by reversing the indices, to denote the same quantity.

The invention in its preferred form therefore consists:

1. In producing the resilient supports and their mounting in such a way that all the coefficients having two different indices in the potential energy, namely:

$$A_{xy}, A_{yz} \ldots A_{\theta\phi}, A_{\phi\psi}$$

are zero. More particularly it consists in utilising for this purpose resilient supports of adhering rubber, that is to say adhering to metal frames, said supports having different rigidities according to the directions, which differences of rigidity will be used as will be explained.

2. In producing moreover the supports and their mounting in such a way that the natural frequencies:

$$\frac{1}{2\eta}\sqrt{\frac{A_{xx}}{m}};\ \frac{1}{2\eta}\sqrt{\frac{A_{yy}}{m}};\ \frac{1}{2\eta}\sqrt{\frac{A_{zz}}{m}}$$

$$\frac{1}{2\eta}\sqrt{\frac{A_{\theta\theta}}{i_1}};\ \frac{1}{2\eta}\sqrt{\frac{A_{\phi\phi}}{i_2}};\ \frac{1}{2\eta}\sqrt{\frac{A_{\psi\psi}}{i_3}}$$

are clearly situated outside the ranges of frequencies where the disturbing causes which can be feared are capable of maintaining a state of vibration.

The whole of these conditions will be called "conditions of decoupling" of the suspension. It can be seen that the invention consists in arranging the supports in such a way that the ellipsoid with six dimensions:

$$V = \text{constant}$$

has its axes of symmetry which coincide in position with those of the ellipsoid with six dimensions

$$T = \text{constant}$$

the ratio of the magnitudes of the corresponding axes of the two ellipsoids (which ratio destroys the natural frequencies of the suspended bodies) being in addition subjected to the qualitative condition that these natural frequencies are clearly outside the ranges of external or disturbing frequencies maintaining the vibrations.

There are likewise certain particular cases of the application of the invention in which one is sure that certain modes of vibrations will not be excited. For example, if the nature of the problem requires that neither the linear vibration z nor the rotations θ and ψ are capable of being excited, it is consequently indifferent from the practical point of view to require or not require that the quantities $A_{\theta z}$, $A_{\psi z}$, $A_{\theta\psi}$ are or are not zero and to submit to any restrictions the ratios

$$\frac{A_{zz}}{m};\ \frac{A_{\theta\theta}}{i_1};\ \frac{A_{\psi\psi}}{i_3}$$

It is therefore to be understood that according to the invention it is possible to neglect the corresponding conditions of decoupling.

It should here be noted that the conditions of decoupling between one of the rotations θ, ϕ, ψ and one of the translations x, y, z must be regarded as known more particularly by the following documents:

(a) "Relations between the suspension of vehicles and their galloping motion" (Y. Rocard), see Le Genie Civil of the 12th and 19th January, 1935 in which article the conditions are given for decoupling in a motor-car the galloping oscillation (rotation about a transverse horizontal axis from the pumping oscillation (vertical movement of the body).

(b) French Patent No. 815,491 (Julien and Paulsen) of the 28th March, 1936, for "Improvements in fixing devices for explosion engines or the like", in which a similar condition is indicated for an aero-engine suspension when it is not possible to put the ring of resilient supports in the transverse plane which contains the centre of gravity.

On the other hand, however, the other conditions of decoupling are not described and their production by means of resilient supports provided with different rigidities according to the directions are essential features of the invention.

It is evident that any resilient support fulfilling the conditions laid down by the invention can be used but preferably use will be made of supports of the types described in the following French Patents: French Patent No. 798,631 of 28th November, 1935, and French Patent No. 817,656 of 15th May, 1936, the invention not being limited thereby.

The resilient properties of these supports, at least for those having a simple geometrical form, have formed the object of a study by Mr. Y. Rocard published in the Journal de Physique of 1937.

In order to appreciate the following description it is sufficient to know that if the rectangural axes ox_i, oy_i, oz_i are considered connected to that one of the frames of the support which is fixed, any displacement Δx_i, Δy_i, Δz_i and any rotation $\Delta\theta_i$, $\Delta\phi_i$, $\Delta\psi_i$ of the other frame engender resilient forces and couples which are characteristic of the geometry of the support.

If, for example, the support consists of a small parallelepipedon of rubber adhering to two parallel metal frames forming two opposite faces of said parallelepipedon, and if the axis ox_i is normal to these frames, with the axes oy_i and oz_i respectively normal to the other faces, the displacements in question bring into action:

(a) A force, of components:

N_{1_i} Δx_i on ox_i
N_{2_i} Δy_i on oy_i
N_{3_i} Δz_i on oz_i

(b) A couple, of components:

T_{1_i} $\Delta\theta_i$ on ox_i
T_{2_i} $\Delta\phi_i$ on oy_i
T_{3_i} $\Delta\psi_i$ on oz_i

and the potential energy for this particular support will be:

$$V_i = N_{1i}\frac{(\Delta x_i)^2}{2} + N_{2i}\frac{(\Delta y_i)^2}{2} + N_{3i}\frac{(\Delta z_i)^2}{2} + T_{1i}\frac{(\Delta\theta_i)^2}{2} + T_{2i}\frac{(\Delta\phi_i)^2}{2} + T_{3i}\frac{(\Delta\psi_i)^2}{2}$$

Knowing how this support is orientated relatively to the system of axes ox_i, oy_i, oz_i connected to the principal axes of inertia of the suspended body, and knowing at what point of the suspended body it is fixed, it is easy to calculate the displacements Δx_i, $\Delta y_i \ldots \Delta\psi_i$ which result, for the frame of said support, from displacements x, y, z, θ, ϕ, ψ, of the body itself.

V_i is thus obtained as a function of x, y, z, θ, ϕ, ψ, which is the contribution of this support No. i to the total potential energy V. The coefficients A_{xy}, $A_{xz} \ldots$ should thus appear in the form:

$$A_{xy} = \Sigma_i(A_{xy})_i \ldots A_{\theta\psi} = \Sigma_i(A_{\theta\psi})_i$$

of similar coefficients defined for each support, and in order to obtain the conditions:

$$A_{xy} = 0 \ldots A_{\theta\psi} = 0$$

there are used according to the invention superabundant means consisting either in eliminating separately the coefficients $(A_{xy})_1$ . . . by a suitable orientation of the supports attached at given points, or in eliminating the aggregates $\Sigma(A_{xy})_1$ . . . by giving a certain symmetry to the arrangement of the points where the supports are attached.

There will now be described by way of example particular embodiments of the invention which will make the nature of the invention still better understood and which will give the desired means for utilising the supports with rigidity variable according to the directions in order to obtain the conditions required by the invention.

Figure 1 shows a front view of a radial aero-engine (the engine not shown) the profile of which is shown in Figure 2 and which must be suspended by a certain number of supports: **1**, **2**, **3**, **4**, to be placed in a plane. In addition for reasons of place and space the supports are necessarily placed at the fixed positions indicated and it is not possible to distribute them symmetrically above and below. Under these conditions it can be seen that if the supports **3** and **4** are identical and symmetrically placed relatively to oz, a vibration θ of the engine is properly decoupled from a vertical vibration z, but that it is not necessarily decoupled from a horizontal vibration x. It will be seen that this decoupling can be attained if the individual supports: **1**, **2**, **3**, **4** have radial rigidities K_1 K_2 K_3 K_4 and tangential rigidities H_1 H_2 H_3 H_4 differing from one another, with supports precisely imposed by the invention.

It is supposed that the radial and tangential directions are axes of symmetry for each support. The supports **3** and **4** are each situated in an angle α as shown on the figure. Under these conditions it is found that the potential energy of the suspension, for movements x and θ alone contemplated, is:

$$=(H_1+H_2+2K_3\cos^2\alpha+2H_3\sin^2\alpha)\frac{x^2}{2}+$$

$$(H_1+H_2+2H_3\frac{(R\theta^2)}{2}+(H_1-H_2+2H_3\sin\alpha)xR\theta$$

As, on the other hand, the main force is simply:

$$T=i\frac{\dot{\theta}^2}{2}+m\frac{\dot{x}^2}{2}$$

(in which formula m=mass of the engine, i= moment of inertia for the rotation θ, R=radius of the circle of supports) and does not contain the right-angle terms, the conditions of decoupling of the movement x and of the movement θ are:

$$H_1-H_2+2H_3\sin\alpha=0 \qquad (1)$$

Thus, according to the invention the support of the bottom will be chosen with a tangential rigidity H_2 greater than H_1 and such that it is equal to $(H_1+2H_3\sin\alpha)$. As, on the other hand, K_1 K_2 K_3 are determined by other conditions, more particularly frequency of vertical oscillation, it can be seen that in order to obtain the relation (1) in all cases, it is necessary to arrange supports which are not identical and for which the ratios:

$$\frac{H_1}{K_1};\ \frac{H_2}{K_2};\ \frac{H_3}{K_3}$$

are exactly determined.

One remarkable case merits special mention: it is that in which the supports of the adhering rubber type work under shear in two directions corresponding to two degrees of freedom, or three degrees if a rotation in this plane is added thereto, which it is necessary to decouple.

In this case the main rigidities K_1 and H_1 in this plane are the same and they are equal to the apparent rigidity of the support for any direction of this plane. If in this case the mounting shown in Figure 1 is again studied and if it is supposed (Figures 3 and 4), for example in this case, for greater simplicity that the supports **1**, **3**, **4** are identical, the condition of decoupling found, which was:

$$H_1-H_2+2H_3\sin\alpha=0$$

becomes:

$$K_2=K_1(1+2\sin\alpha)$$

consequently the proportioning of the support of rigidities

$$\frac{K_2}{K_1}$$

could always be sufficient to ensure the coupling whatever the position previously imposed of the supports **3**, **4**.

Another characteristic case is that of Figures 5 and 6 which show a side view and plan view of a motor vehicle engine **5** resting on three resilient supports **1**, **3**, **3'** for example. This engine transmits the rotation with oscillations of couple to a horizontal axis G_h and its principal axes of inertia G_x, G_z are inclined. Consequently the oscillations of couple of horizontal axis result not only in an oscillation about G_x (component G_a of the couple forming a rolling motion) but also in an oscillation about oz (component of couple G_b zig-zag couple). There are also to be considered pitching oscillations which result from the fact that the alternate forces applied by the operation itself of the engine do not pass through the centre of gravity G.

According to the invention the engine is fixed on only three resilient supports: **1**, **3**, **3'**; the support **1** being situated on the axis G_x to the rear, **3** and **3'** being situated in front of the centre of gravity G. The exact position of 1 is determined in a known manner, for example as indicated in French Patent No. 750,029 of the 16th April, 1932, and French Patent No. 782,906 of the 9th March, 1934, in such a way that one is situated at the centre of percussion corresponding to the alternate forces applied to the engine. According to the Franch patents just cited, however, it is only necessary to give to said support an elasticity in the vertical direction. According to the invention, on the contrary, it is intended to decouple the zig-zag oscillations and the rolling oscillations (which are necessarily caused in the case in question) from the oscillations of translation and also from one another for the purpose of more easily maintaining the frequencies of resonance of all these movements in a dangerous range. The identity and the symmetry of the position of the supports **3** and **3'** already ensures this result for rolling motion. The determination of the rigidity of these supports in the sense of the rolling motion also raises the technique known in France under the name "Floating Engine". In order to obtain the decoupling of the zig-zag motion horizontal rigidities (plane xGy) K_1 H_1 and K_3 H_3 are therefore given to the supports of adhering rubber preferred by the invention such that the natural oscillation of zig-zag motion takes place about G.

For this the formula of the potential energy V for movements x, y, θ, of the engine is written by supposing that the rigidities K_1 H_1, K_3 H_3 are the principal rigidities of their respective supports.

It is found that this potential energy is:

$$V=(K_1+2K_3\sin^2\alpha+2H_3\cos^2\alpha)\frac{x^2}{2}+$$

$$(H_1+2K_3\cos^2\alpha+2H_3\sin^2\alpha)\frac{y^2}{2}+$$

$$(H_1d_1^2+2H_3d_3^2\sin)\frac{2}{2}+(H_1d_1-2H_3d_3\sin\alpha)y\theta$$

As the motion of the body is referred to its principal axes of inertia, the condition for which there is decoupling between y and θ is thus simply:

$$H_1d_1=2H_3d_3\sin\alpha$$

It can be seen that the radial rigidities are not made to intervene and that the coupling of the movement x and of the movement θ are prevented by the symmetries of the problem treated.

Of course, the problem would be of the same kind if more than three supports, for example four supports, were provided. It can also be supposed that the supports are not situated exactly in the plane xGy, but that they are situated in the position inclined to the vertical at a level below the plane. If the principal rigidities of the supports are considered in projection on the plane xGy the same problems will always occur the solutions of which are sufficiently indicated by the three examples which have been described.

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